EXERCISES [MAI 5.17]

KINEMATICS

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. (a) (i) $v = \frac{ds}{dt} = 12 - 3t^2$. (ii) For $t=1$, $v = 9ms^{-1}$

(b) (i) $a = \frac{dv}{dt} = -6t$. (ii) For $t=1$, $a = -6ms^{-2}$

(c) (i) $v = -15 \Leftrightarrow 12 - 3t^2 = -15 \Leftrightarrow 3t^2 = 27 \Leftrightarrow t^2 = 9 \Leftrightarrow t = 3$ sec

(ii) Stationary $\Rightarrow v = 0 \Leftrightarrow 3t^2 - 12 = 0 \Leftrightarrow t^2 = 4 \Leftrightarrow t = 2$ sec

(iii) At point A $\Rightarrow s = 0 \Leftrightarrow 12t - t^3 = 0 \Leftrightarrow t(12 - t^2) \Leftrightarrow t = 0$ or $t = \sqrt{12} = 2\sqrt{3}$ sec

2. (a) $v = \int a dt = -3t^2 + c$

When $t = 0, v = 12 \Rightarrow 0 + c = 12 \Leftrightarrow c = 12$. Hence, $v = -3t^2 + 12$

(b) $s = \int v dt = -t^3 + 12t + c$

When $t = 0, s = 0 \Rightarrow 0 + c = 0 \Leftrightarrow c = 0$. Hence, $s = -t^3 + 12t$

(c) Maximum displacement $\Rightarrow \frac{ds}{dt} = 0 \Rightarrow v = 0$ (car stationary)

$v = 0 \Leftrightarrow 3t^2 - 12 = 0 \Leftrightarrow t^2 = 4 \Leftrightarrow t = 2$ sec

Then $s = -2^3 + 12 \times 2 = 16m$

(d) $v = -3t^2 + 12 > 0 \Leftrightarrow 12 > 3t^2 \Leftrightarrow t^2 < 4 \Leftrightarrow 0 < t < 2$

(e) $s = -t^3 + 12t > 0$. By observing the corresponding graph $0 < t < 2\sqrt{2}$

3. (a) $a = \frac{dv}{dt} = -6t$.

(b) $s = \int v dt = -t^3 + 12t + c$

When $t = 0, s = 0 \Rightarrow 0 + c = 0 \Leftrightarrow c = 0$. Hence, $s = -t^3 + 12t$

(c) Stationary $\Rightarrow v = 0 \Leftrightarrow 3t^2 - 12 = 0 \Leftrightarrow t^2 = 4 \Leftrightarrow t = 2$ sec

(d) (i) The displacement after 3 seconds is $s = 3^3 - 12 \times 3 = -9m$

(ii) For the distance travelled,

METHOD A: By GDC: $d = \int_0^3 |v| dt = \int_0^3 |3t^2 - 12| dt = 23m$

METHOD B: Without GDC: the direction changes at $t = 2$.

$$\int_0^2 v dt = \int_0^2 (3t^2 - 12) dt = \left[t^3 - 12t\right]_0^2 = (8 - 24) - 0 = -16m$$

$$\int_2^3 v dt = \int_2^3 (3t^2 - 12) dt = \left[t^3 - 12t\right]_2^3 = (27 - 36) - (8 - 24) = -9 + 16 = 7m$$

Hence, $d = 16m + 7m = 23m$.

(e) Two shaded regions: from 0 to 2 (negative), from 2 to 3 (positive).

4. (a) $v = \dfrac{ds}{dt} = 10 - t$

When $t = 0$, $v = 10$ (m s^{-1})

(b) The velocity is zero when $\dfrac{ds}{dt} = 0 \Leftrightarrow 10 - t = 0 \Leftrightarrow t = 10$ (secs)

(c) $s = 50$ (metres)

5. (a) $S_{min} = 6.05$ (accept (1, 6.05))

(b) $\dfrac{ds}{dt} = -15 \sin 3t + 2t$

$a = \dfrac{d^2s}{dt^2} = -45 \cos 3t + 2$

(c) **EITHER**

Maximum value of a when $\cos 3t$ is minimum *ie* $\cos 3t = -1$

OR

At maximum $\dfrac{da}{dt} = 0$ $(135 \sin 3t = 0)$

$t = \dfrac{\pi}{3}$ (1.05 also accepted)

6. (a) Evidence of using $a = \dfrac{dv}{dt} = 3e^{3t-2}$

$a(1) = 3e$ $(= 8.15)$

(b) solve $e^{3t-2} = 22.3 \Leftrightarrow t = 1.70$

(c) using $s = \int v dt \int e^{3t-2}\, dt = \dfrac{1}{3}\left[e^{3t-2}\right]$

Distance travelled $= \dfrac{1}{3}\left[e^{3t-2}\right]_0^1 = \dfrac{1}{3}\left(e^1 - e^{-2}\right)$ $\left[= \dfrac{1}{3}\left(e - e^{-2}\right) = 0.861\right]$

7. (a) $s = t^4 - t^2 + c$

$8 = 2^4 - 2^2 + c \Leftrightarrow c = -4$

$s = t^4 - t^2 - 4$

(b) $a = 12t^2 - 2,$

For $t=1$, $a = 10ms^{-2}$

8. $s = \int v\, dt$ $s = \dfrac{1}{2}e^{2t-1} + c$

Substituting $t = 0.5$ $\dfrac{1}{2} + c = 10 \Rightarrow c = 9.5$

$s = \dfrac{1}{2}e^{2t-1} + 9.5$

Substituting $t = 1$ $s = \dfrac{1}{2}e + 9.5 (= 10.9 \text{ to } 3\, s.f.)$

9. $s = -0.5\, e^{-2t} + 6t^2 + c$

Substituting $t = 0$, $s = 2 \Rightarrow c = 2.5$

$s = -0.5\, e^{-2t} + 6t^2 + 2.5$

10. $s = \int \left(6e^{3x} + 4\right)dx = 2e^{3t} + 4t + C$

substituting $t = 0$, $7 = 2 + C \Rightarrow \quad C = 5$

$s = 2e^{3t} + 4t + 5$

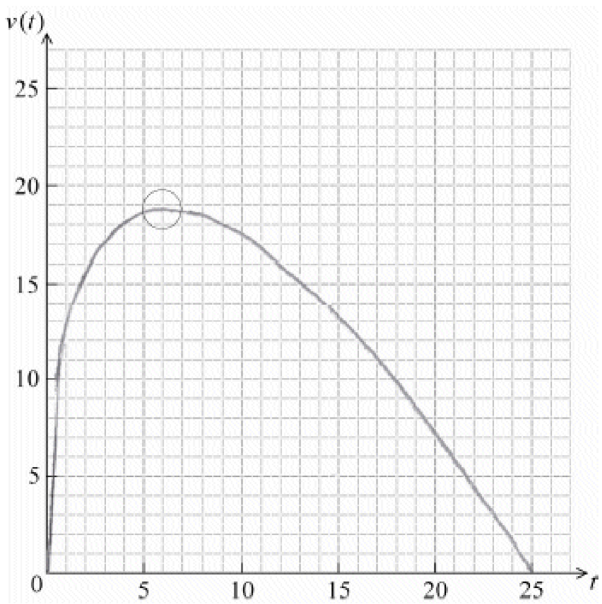
11. (a) $a = \dfrac{dv}{dt} = -10$

(b) $s = \int v dt = 50t - 5t^2 + c$

$40 = 50(0) - 5(0) + c \Rightarrow c = 40$

$s = 50t - 5t^2 + 40$

12. (a)



(b) (i) $d = \ d = \int_0^9 v dt = \int_0^9 (15\sqrt{t} - 3t) dt$,

(ii) $d = 148.5$ (m) (or 149 to 3 sf)

13. $v = \int \left(\dfrac{1}{t} + 3\sin 2t\right) dt = \ln t - \dfrac{3}{2} \cos 2t + c$

substituting (1, 0): $0 = \ln 1 - \dfrac{3}{2} \cos 2 + c \Leftrightarrow c = -0.624$ (or $\dfrac{3}{2}\cos 2$)

$v = \ln t - \dfrac{3}{2}\cos 2t - 0.624 \left(= \ln t - \dfrac{3}{2}\cos 2t + \dfrac{3}{2}\cos 2 \right.$

$v(5) = 2.24$ (or exact answer $\ln 5 - 1.5 \cos 10 + 1.5 \cos 2$)

14. (a)

Function	**Graph**
displacement	A
acceleration	B

(b) $t = 3$

15. (a) $s = 25t - \frac{4}{3}t^3 + c$

Substituting $s = 10$ and $t = 3$: $10 = 25 \times 3 - \frac{4}{3}(3)^3 + c \Leftrightarrow \quad c = -29$

$s = 25t - \frac{4}{3}t^3 - 29$

(b) **METHOD 1**

s is a maximum when $v = \frac{\mathrm{d}s}{\mathrm{d}t} = 0$

$25 - 4t^2 = 0 \Rightarrow t^2 = \frac{25}{4} \Rightarrow t = \frac{5}{2}$

METHOD 2

By GDC graph: max when $t = 2.5$

(c) $25t - \frac{4}{3}t^3 - 29 > 0$ (accept equation)

$m = 1.27$, $n = 3.55$

16. (a) $s = 50t = 10t^2 + 1000$

$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 50 - 20t$

(b) Displacement is max when $v = 0$,

ie when $t = \frac{5}{2}$.

Substituting $t = \frac{5}{2}$, $s = 50 \times \frac{5}{2} - 10 \times \left(\frac{5}{2}\right)^2 + 1000$

$s = 1062.5$ m (or directly by GDC graph, maximum)

(c) $a = -20\ ms^{-2}$

17. Given $s = 40t + 0.5at^2$, then the maximum height is reached when $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$

$at + 40 = 0$

$a = \frac{-40}{25} = -1.6$ (units not required)

18. $s = \int (3t^2 - 4t + 2)dt = t^3 - 2t^2 + 2t + c$

$s(0) = -3 \Rightarrow c = -3$

$s = t^3 - 2t^2 + 2t - 3$

$s = 0 \Leftrightarrow t^3 - 2t^2 + 2t - 3 = 0 \Leftrightarrow t = 1.81$ (sec)

19. $a(t) = -\frac{1}{20}t + 2 \qquad v(t) = -\frac{1}{40}t^2 + 2t + c$

$v = 0$ when $t = 0$, and so $c = 0$

Thus, $v(t) = -\frac{1}{40}t^2 + 2t = -\frac{1}{40}t(t - 80)$.

Since $v(t) \geq 0$ for $0 \leq t \leq 80$, the distance travelled $= \int_0^{60} v(t)\mathrm{d}t$

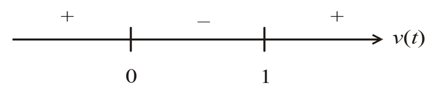
$$= \left[-\frac{1}{120}t^3 + t^2\right]_0^{60} = 60^2\left(1 - \frac{1}{2}\right) = 1800 \text{ m.}$$

20. (a) $s=\int_0^2 (6t^2-6t)dt=\left[2t^3-3t^2\right]_0^2=16-12=4$ m

(b) Distance travelled $=\int_0^2 \left|6t^2-6t\right|dt=6$ m

OR analytically

$v(t) = 6t^2 - 6t = 6t\,(t - 1)$



Distance travelled $= -\int_0^1\left(6t^2-6t\right)\mathrm{d}t+\int_1^2\left(6t^2-6t\right)\mathrm{d}t$

$= -[2t^3-3t^2]_0^1 + [2t^3-3t^2]_1^2 = -(-1)+5 = 6$ m.

(c) There is a change in direction at $t=2$: 1 m backwards, 5 m forward, hence

Displacement = 4m Distance travelled = 6m

21. Total distance $= k\int_0^a \mathrm{e}^{-t/2}\mathrm{d}t = -2k[\mathrm{e}^{-t/2}]_0^a = -2k(\mathrm{e}^{-a/2}-1)$ metres

22. (a) Distance $=\int_0^1 \frac{\mathrm{d}t}{2+t^2} = 0.435$

(b) Acceleration $=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{-2t}{\left(2+t^2\right)^2}$

23. (a) $v=0 \Rightarrow t=\pi$ (first time)

(b) (i) Displacement $=\int_0^{2\pi} e^{-\sqrt{t}}\sin t dt=0.387$

(ii) Distance travelled $=\int_0^{2\pi}\left|e^{-\sqrt{t}}\sin t\right|dt=0.852$

OR more analytically $\int_0^{\pi}\mathrm{e}^{-\sqrt{t}}\sin t\mathrm{d}t+\left|\int_{\pi}^{2\pi}\mathrm{e}^{-\sqrt{t}}\sin t\mathrm{d}t\right| = 0.620 + 0.232 = 0.852$

24. (a) $v(t) = t\sin\left(\frac{\pi}{3}t\right) = 0$ when $t = 0$, $t = 3$ or $t = 6$

(b) (i) The required distance, $d=\int_0^6\left|t\sin\left(\frac{\pi}{3}t\right)\right|\mathrm{d}t$. (ii) $d = 11.5$ m.

OR more analytically

(i) The required distance, $d=\int_0^3 t\sin\left(\frac{\pi}{3}t\right)\mathrm{d}t-\int_3^6 t\sin\left(\frac{\pi}{3}t\right)\mathrm{d}t$

(ii) $d = 2.865 + 8.594 = 11.459 = 11.5$ m.

25. (a) $a=10$ ms^{-2}

(b) $a=100s$

26. (a) $a=2t+10e^t$

(b) $a=(2s+10e^s)v=(2s+10e^s)(s^2+10e^s)$

27. Given $v=\frac{(3s+2)}{(2s-1)}$ then $a = v\frac{\mathrm{d}v}{\mathrm{d}s}=\frac{3(2s-1)-2(3s+2)}{(2s-1)^2}\times\frac{(3s+2)}{(2s-1)} \Leftrightarrow a=\frac{-7(3s+2)}{(2s-1)^3}$

therefore when $s = 2$, $a=\frac{-56}{27}$

B. Paper 2 questions (LONG)

28. (a) $a = -8e^{2t}$,

At $t = \ln 3$ $a = -8e^{2\ln 3} = -72$ ms^{-2}

(b) $s = \int 16 - 4e^{2t} dt = 16t - 2e^{2t} + c$

$s(0) = 0 \Rightarrow c = 2$. Therefore $s = 16t - 2e^{2t} + 2$

(c) $v = 0 \Leftrightarrow 16 - 4e^{2t} = 0 \Leftrightarrow t = \ln 2$

(d) $s = 32\ln 2 - 30$ m,

distance travelled = $\int_0^{\ln 4} |16 - 4e^{2t}|\, dt =$ 18 m

OR $= \int_0^{\ln 2} (16 - 4e^{2t})dt - \int_{\ln 2}^{\ln 4} (16 - 4e^{2t})dt =$ 18m

29. (a) $v = \frac{1}{4}\cos\left(\frac{t}{4}\right)$ $a = -\frac{1}{16}\sin\left(\frac{t}{4}\right)$

(b) $s_{\max} = 1$

(c) $s = 0$ at $t = 0, 4\pi, 8\pi, \ldots$ (i.e. $t = 4k\pi,\ k \in Z^+$)

(d) $\pm\frac{1}{4}$ ms^{-1}

(e) $v = 0$ at $t = 2\pi, 6\pi, 10\pi, \ldots$ (i.e. $t = 2\pi + 4k\pi,\ k \in Z^+$)

(f) $\pm\frac{1}{16}$ ms^{-2}

30. (a) (i) $v(0) = 50 - 50e^0 = 0$ (ii) $v(10) = 50 - 50e^{-2} = 43.2$

(b) (i) $a = \frac{dv}{dt} = -50(-0.2e^{-0.2t}) = 10e^{-0.2t}$ (ii) $a(0) = 10e^0 = 10$

(c) (i) $t \to \infty \Rightarrow v \to 50$

(ii) $t \to \infty \Rightarrow a \to 0$

(iii) when $a = 0$, v is constant at 50

(d) (i) $y = \int v dt = 50t - \frac{e^{-0.2t}}{-0.2} + k = 50t + 250e^{-0.2t} + k$

(ii) $0 = 50(0) + 250e^0 + k = 250 + k \Rightarrow k = -250$

(iii) Solve $250 = 50t + 250e^{-0.2t} - 250 \Rightarrow t = 9.207$ s

31. (a) $\frac{ds}{dt} = 30 - at \Rightarrow s = 30t - a\frac{t^2}{2} + C$

$t = 0 \Rightarrow s = 30(0) - a\frac{(0^2)}{2} + C = 0 + C \Rightarrow C = 0$. Thus $s = 30t - \frac{1}{2}at^2$

(b) (i) vel = 30 – 5(0) = 30m s^{-1}

(ii) Train will stop when $0 = 30 - 5t \Rightarrow t = 6$

Distance travelled = $30t - \frac{1}{2}at^2 = 30(6) - \frac{1}{2}(5)(6^2) = 90$m

90 < 200 => train stops before station.

(c) (i) $0 = 30 - at \Rightarrow t = \frac{30}{a}$

(ii) $30\left(\frac{30}{a}\right) - \frac{1}{2}(a)\left(\frac{30}{a}\right)^2 = 200 \Rightarrow \frac{900}{a} - \frac{450}{a} = \frac{450}{a} = 200 \Rightarrow a = \frac{9}{4} = 2.25$ ms^{-2}

32. (a) When $t = 0$, $h = 2 + 20 \times 0 - 5 \times 0^2 = 2 \Rightarrow h = 2$

(b) When $t = 1$, $h = 2 + 20 \times 1 - 5 \times 1^2 = 17$

(c) (i) $h = 17 \Rightarrow 17 = 2 + 20t - 5t^2$

(ii) $5t^2 - 20t + 15 = 0 \Leftrightarrow 5(t^2 - 4t + 3) = 0 \Leftrightarrow \ t = 3$ or 1

(d) (i) $h = 2 + 20t - 5t^2 \Rightarrow \dfrac{dh}{dt} = 0 + 20 - 10t = 20 - 10t$

(ii) $t = 0 \Rightarrow \dfrac{dh}{dt} = 20 - 10 \times 0 = 20$

(iii) $\dfrac{dh}{dt} = 0 \Leftrightarrow 20 - 10t = 0 \Leftrightarrow t = 2$

(iv) $t = 2 \Rightarrow h = 2 + 20 \times 2 - 5 \times 2^2 = 22 \Rightarrow h = 22$

33. (a) (i) $t = 0$ $s = 800$

$t = 5$ $s = 800 + 500 - 100 = 1200$

distance in first 5 seconds = 1200 – 800 = 400 m

(ii) $v = \dfrac{ds}{dt} = 100 - 8t$

At $t = 5$, velocity = 100 – 40= 60 m s^{-1}

(b) (i) Velocity = 36 m s^{-1} $\Rightarrow 100 - 8t = 36$

$t = 8$ seconds after touchdown.

(ii) When $t = 8$, $s = 800 + 100(8) - 4(8)^2 = 800 + 800 - 256 = 1344$ m

(c) If it touches down at P, it has 2000 – 1344 = 656 m to stop.

To come to rest, $100 - 8t = 0 \Rightarrow t = 12.5$ s

Distance covered in 12.5 $s = 100(12.5) - 4(12.5)^2 = 625$

Since 625 < 656, it can stop safely.

34. (a) (i) At release(P), $t = 0$, $s = 48 + 10 \cos 0 = 58$ cm below ceiling

(ii) $58 = 48 + 10 \cos 2\pi t \Leftrightarrow \cos 2\pi t = 1$ $t = 1$sec

(b) (i) $\dfrac{ds}{dt} = -20\pi \sin 2\pi t$

(ii) $v = \dfrac{ds}{dt} = -20\pi \sin 2\pi t = 0 \Leftrightarrow \sin 2\pi t = 0 \Leftrightarrow t = 0, \dfrac{1}{2}$ ... (at least 2 values)

$s = 48 + 10 \cos 0$ **or** $s = 48 + 10 \cos \pi$

= 58 cm (at P) = 38 cm (20 cm above P)

Note: *May be deduced from recognizing that amplitude is 10.*

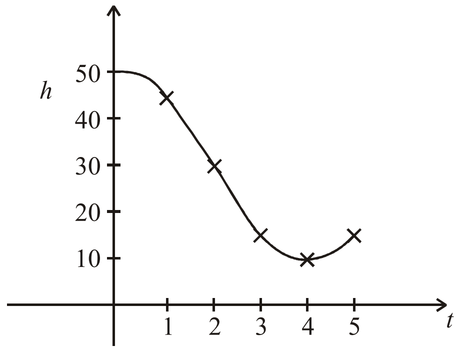
(c) $48 + 10 \cos 2\pi t = 60 + 15 \cos 4\pi t \Leftrightarrow t = 0.162$ secs

(d) 12 times

35. (a) $t = 2 \Rightarrow h = 50 - 5(2^2) = 50 - 20 = 30$

OR $h = 90 - 40(2) + 5(2^2) = 30$

(b)



(c) (i) $\frac{dh}{dt} = \frac{d}{dt}(50 - 5t^2) = -10t$

(ii) $\frac{dh}{dt} = \frac{d}{dt}(90 - 40t + 5t^2) = -40 + 10t$

(d) When $t = 2$ (i) $\frac{dh}{dt} = -10(2) = -20$ **or** (ii) $\frac{dh}{dt} = -40 + 10 \times 2 = -20$

(e) $\frac{dh}{dt} = 0 \Rightarrow -10t = 0\ (0 \le t \le 2)$ **or** $-40 + 10t = 0 (2 \le t \le 5)$

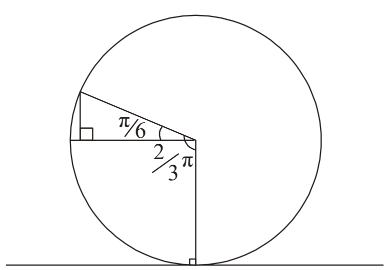
$t = 0$ $t = 4$

(f) When $t = 4$, $h = 90 - 40(4) + 5(4^2) = 90 - 160 + 80 = 10$

36. (a) arc AB = $\boldsymbol{r}\theta = 5\pi/2 = 7.85$ (m)

(b) Area of sector AOB $A = \frac{1}{2}r^2\theta = 58.9$ (m^2)

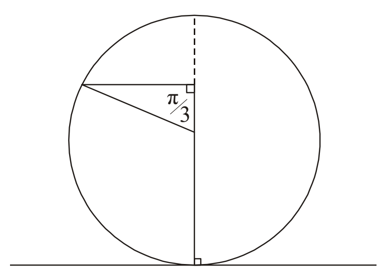
(c) **METHOD 1**



angle = $\frac{\pi}{6}$ $(30°)$

height = $15 + 15 \sin\frac{\pi}{6} = 22.5$ (m)

METHOD 2



angle = $\frac{\pi}{3}$ $(60°)$

height = $15 + 15 \cos\frac{\pi}{3} = 22.5$ (m)

(d) (i) $h\left(\frac{\pi}{4}\right) = 15 - 15\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) = 25.6$ (m)

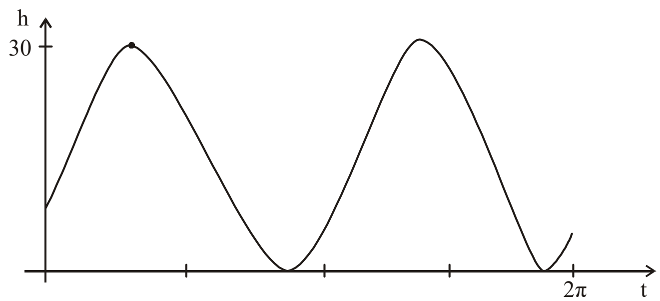
(ii) $h(0) = 15 - 15\cos\left(0 + \frac{\pi}{4}\right) = 4.39$(m)

(iii) **METHOD 1**

Highest point when $h = 30$

$30 = 15 - 15\cos\left(2t + \frac{\pi}{4}\right) \Leftrightarrow \cos\left(2t + \frac{\pi}{4}\right) = -1 \Leftrightarrow t = 1.18 \left(\text{accept } \frac{3\pi}{8}\right)$

METHOD 2



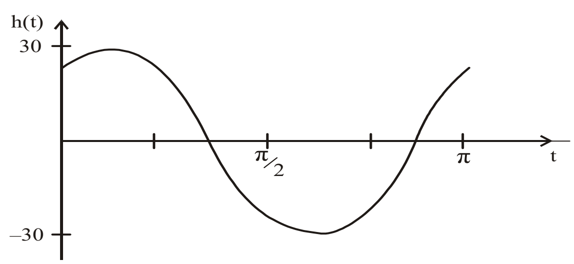
Sketch of graph of h, $t = 1.18$

METHOD 3

$h'(t) = 0 \Leftrightarrow \sin\left(2t + \frac{\pi}{4}\right) = 0 \Leftrightarrow t = 1.18 \left(\text{accept } \frac{3\pi}{8}\right)$

(e) $h'(t) = 30\sin\left(2t + \frac{\pi}{4}\right)$

(f) (i)



(ii) **METHOD 1**
Maximum on graph of h'
$t = 0.393$

METHOD 2
Minimum on graph of h'
$t = 1.96$

METHOD 3
Solving $h''(t) = 0$

One or both correct answers: $t = 0.393$, $t = 1.96$

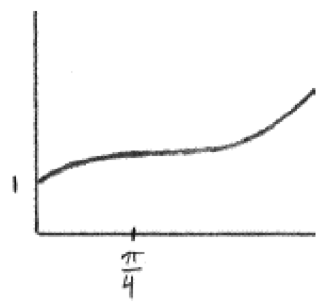
37. (a) $v = 1$

(b) (i) $\frac{dv}{dt} = 0 \Leftrightarrow 2 - 2\sin 2t = 0 \Leftrightarrow \sin 2t = 1 \Leftrightarrow 2t = \frac{\pi}{2} \Leftrightarrow t = \frac{\pi}{4}$

$k = \frac{\pi}{4}$

(ii) $v = 2\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{4}\right) = \frac{\pi}{2}$

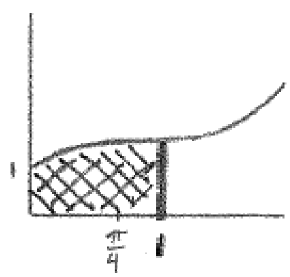
(c)



Note
y-intercept at (0, 1),
zero gradient at t $= \frac{\pi}{4}$,
concave down to the left of $\frac{\pi}{4}$ **and**
concave up to the right of $\frac{\pi}{4}$

(d) (i) $d = \int_0^1 (2t + \cos 2t)\,dt$, **OR** $\left[t^2 + \frac{\sin 2t}{2}\right]_0^1$, $1 + \frac{\sin 2}{2}$

(ii)



Note: The line at $t = 1$ needs to be clearly after $t = \frac{\pi}{4}$.

38. (a) $f(x) = -10(x + 4)(x - 6)$

(b) $x = (-4+6) / 2 = 1$

$y = -10(1+4)(1-6) = 250$ vertex (1, 250)

$f(x) = -10(x-1)^2 + 250$

(c) simplify $-10(x + 4)(x - 6) = -10(x^2 - 6x + 4x - 24) = 240 + 20x - 10x^2$

OR $-10(x-1)^2 + 250 = -10(x^2 - 2x + 1) + 250 = 240 + 20x - 10x^2$

(d) (i) vertex of parabola, $v'(t) = 0$

$t = 1$

(ii) $a(t) = v'(t) = 20 - 20t$

speed is zero $\Rightarrow t = 6$, $a(6) = -100$ (m s^{-2})

39. (a) (i) $s = \int (40 - at)\mathrm{d}t = 40t - \frac{1}{2}at^2 + c$

substituting $s = 100$ when $t = 0$ ($c = 100$)

$s = 40t - \frac{1}{2}at^2 + 100$

(ii) $s = 40t - \frac{1}{2}at^2$

(b) (i) stops at station, so $v = 0$ when $t = \frac{40}{a}$ (seconds)

(ii) substituting $t = \frac{40}{a}$ to the formula for s from (a) (ii)

$40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2} = 500 \Leftrightarrow \frac{1600}{a} - \frac{800}{a} = 500 \Leftrightarrow \frac{800}{a} = 500$

$\Leftrightarrow a = \frac{8}{5}$

(c) **METHOD 1**

$v = 40 - 4t$, stops when $v = 0 \Leftrightarrow 40 - 4t = 0 \Leftrightarrow t = 10$

OR from (b) $t = \frac{40}{4} = 10$

Then

$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2 = 200$

since 200 < 500 train stops before the station

METHOD 2

a is deceleration

$4 > \frac{8}{5}$

so stops in shorter time, so less distance travelled, so stops before station

40. (a) $v = \int a\,dt = \int -\frac{1}{(1+t)^2}\,dt = \frac{1}{1+t} + c$

When $t = 1, v = 8 \Rightarrow \frac{1}{2} + c = 8 \Rightarrow c = \frac{15}{2}$

Hence, $v = \frac{1}{1+t} + \frac{15}{2}$

When $t = 3, v = \frac{1}{4} + \frac{15}{2} = \frac{31}{4}$ m s^{-1}

(b) When $t \to \infty, v \to \frac{15}{2}$ ms^{-1}

(c) $s = \int v\,dt = \int \left(\frac{1}{1+t} + \frac{15}{2}\right)dt = \left[\ln(1+t) + \frac{15}{2}t\right]_1^3$

$= \ln 4 + \frac{45}{2} - \ln 2 - \frac{15}{2} = \ln 2 + 15$ m

41.

(a) maximum when $\frac{dv}{dt} = 0$ (or any correct method)

$t = 3$

$t = 3$, $\frac{d^2 v_A}{dt^2} = -1 \Rightarrow$ maximum

$\Rightarrow$ maximum $v_A = 6$ (ms^{-1})

(b) Using acceleration $= \frac{dv}{dt}$

$= \frac{1}{5} e^{0.2t}$

when $t = 4$, $a = 0.445$ (ms^{-2})

(c) using $s_A = \int v_A \, dt$ or $s_B = \int v_B \, dt$

$s_A = -\frac{1}{6}t^3 + \frac{3}{2}t^2 + \frac{3}{2}t + c$

when $t = 0$, $s_A = 0 \Rightarrow c = 0$

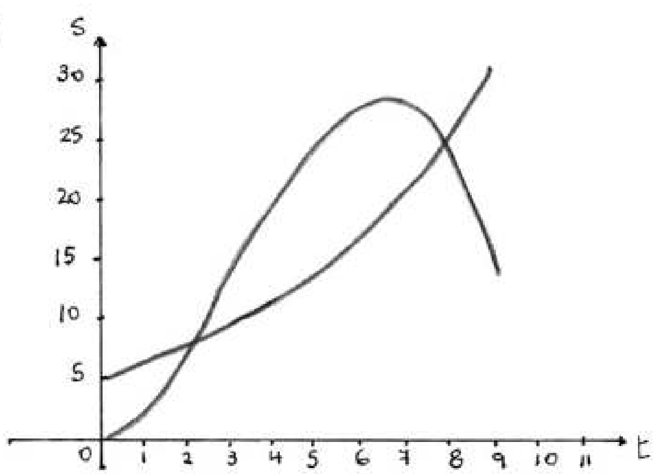
$s_B = \int e^{0.2t} \, dt = 5e^{0.2t} + d$

when $t = 0$, $s_B = 5 \Rightarrow d = 0$

$s_A = -\frac{1}{6}t^3 + \frac{3}{2}t^2 + \frac{3}{2}t$

$s_B = 5e^{0.2t}$

(d) (i)



(ii) $t = 1.95$ and $t = 7.81$